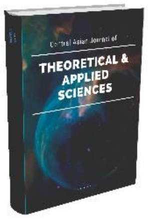

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Fixed Point Theories and Their Feasibility for Teaching in Middle School

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Abstract: *The field of "Fixed Point Theories in Mathematics" is an important and intriguing area for study in the realm of mathematics, which warrants inclusion in middle school curricula. This subject encompasses mathematical concepts and ideas related to fixed points in mathematics, which play a vital role in numerous scientific and technological applications.*

A comprehensive understanding of fixed point theories makes a significant contribution to the development of logical thinking and mathematical reasoning skills among students. This specialization provides them with an opportunity to explore the world of mathematics in an enjoyable and practical manner. Proficiency in the principles of fixed points enhances problem-solving skills, fosters creative thinking, and encourages critical thinking.

A study was conducted at Al-Shamoukh School in Al-Rusafa, during the 2022-2023 academic year. One hundred female students were randomly selected and divided into two groups: an experimental group that studied Fixed Point Theories, and a control group that followed the standard curriculum. Monthly test performance was compared, revealing that students in the experimental group achieved significantly better results than their counterparts in the control group. The success rate in the experimental group reached 100%, compared to 90% in the control group. Moreover, the percentage of students scoring high grades increased in the experimental group.

Based on these results, the study concluded that teaching the topic of Fixed Point Theories positively impacted the performance and understanding of female students in mathematics. The study recommended enhancing the curriculum by incorporating this topic and raising awareness among students and teachers regarding its importance. It also suggested improving teacher training in the delivery of this subject.

Keywords: *Fixed Point Theories, Middle School, Mathematics Education.*

INTRODUCTION:

The topic of "Fixed Point Theorems in Mathematics" is an important and intriguing mathematical field deserving inclusion in middle school education. This subject delves into mathematical ideas and concepts related to fixed points in mathematics, which play a fundamental role in numerous scientific and technological applications.

Understanding fixed point theories significantly contributes to developing logical thinking and mathematical reasoning skills among students. This subject provides them with an opportunity to explore the world of mathematics in an enjoyable and beneficial manner. Grasping the principles of fixed points enhances problem-solving abilities and fosters creative thinking.

The presentation of Fixed Point Theorems in middle school can be done interactively, tailored to the students' level. Simple examples and illustrations can be used to explain the fundamental concepts. Furthermore, interactive games and activities can be employed to make the lesson more engaging and enjoyable.

For instance, the concept of a fixed point can be introduced using simple examples such as the "magic spinner game," where a constant number remains fixed on the spinner regardless of the player's moves.

In summary, Fixed Point Theorems in mathematics is a topic that can be taught in an enjoyable and interactive manner in middle school. It contributes to enhancing students' mathematical skills and logical thinking, serving as a gateway to explore deeper realms of mathematics and sciences.

IMPORTANCE OF THE RESEARCH:

Studying this subject enhances fundamental mathematical skills such as algebra, geometry, and mathematical reasoning. When taught effectively, it can aid students in grasping deeper mathematical concepts. Fixed point theories require logical thinking and strong reasoning, encouraging better problem-solving skills and fostering logical application. These theories also enhance students' abilities for self-discovery and self-learning. In other words, students can independently use the concepts learned in this field to explore new patterns and details. Furthermore, fixed point theories can be beneficial in various real-life applications. For instance, they are used in computer science to solve search problems and numerical analysis and in engineering to solve control and design problems. If introduced in an exciting and engaging manner, the topic of fixed point theories can increase students' interest in mathematics and motivate them to continue studying it in the future.

Overall, fixed point theories in mathematics are an important subject that can contribute to developing students' mathematical skills and logical thinking, opening doors to a deeper understanding of mathematics and its real-world applications.

RESEARCH OBJECTIVES:

1. Shed light on the topic of fixed point theorems in mathematics.
2. Determine the feasibility of teaching the subject by selecting a sample and teaching them the topic, then evaluating the progress of this group in mathematics.
3. Identify the scientific development in the field of mathematics for the experimental group.

THEORETICAL FRAMEWORK OF THE RESEARCH:

Fixed point theories constitute a set of mathematical theories that focus on studying points unaffected by the application of a mathematical operation or function. These points are termed "fixed points" because they remain stationary or unchanged when a function is applied to them. Fixed point theories are highly significant in mathematics and quantitative sciences, with numerous applications across various fields[i] ..

1. "There are several theories and concepts associated with fixed points, among the most famous are:
2. Banach Fixed-Point Theorem: Originating from linear algebra, this theorem states the existence of at least one fixed point for any contraction mapping.

3. Picard's Fixed-Point Theorem: Used in engineering, it proves the existence of a fixed point for any mapping from a space to itself.
4. Fixed-Point Theory in Dynamics: Employed in studying dynamical systems and temporal transformations in natural sciences and engineering.
5. Banach-Tarski Paradox for Fixed Points: Used in analyzing complex problems in mathematics and computer science[(ii)] ".

Banach's Fixed-Point Theorem belongs to the realm of linear algebra, focusing on studying points that remain unchanged under a linear transformation. This theorem expresses a fundamental idea in mathematics, stating that if you have a linear transformation from a linear space to itself, there will be at least one fixed point under this transformation. Simply put, this means there exists a point that doesn't change under the influence of the linear transformation..[(iii)]

Banach's Fixed-Point Theorem addresses the existence of at least one fixed point under the influence of a linear transformation within a linear space.

Let's suppose we have a linear transformation represented by the function T, which takes a point x in the linear space and maps it to a new point, T(x). If the transformation T takes points from the linear space back to itself, meaning $T: V \rightarrow V$, where V is the linear space, then we say that the point x in V is a fixed point under the transformation T if it satisfies the following equation:

$T(x) = x$

In simpler terms, if x is a fixed point under T, it means it remains unchanged when the transformation T is applied to it.

Banach's Theorem proves the existence of this fixed point, relying on the concepts of natural mapping and optimality in linear algebra. It can be conceptualized as follows:

If T is a linear transformation from V to itself, and we have a certain arrangement of points x in V, then either we already have a fixed point (i.e., an x that satisfies T(x) = x), or we can find a new fixed point by performing appropriate linear algebraic operations. This means that there is always at least one fixed point for any linear transformation from the linear space to itself[(iv)]..

Let's translate the example of Banach's Fixed-Point Theorem applied to a linear algebra scenario concerning an electrical engineering model describing an Alternating Current (AC) system in an electrical circuit using a matrix representation.

Assume we have a simple model used in electrical engineering to describe an AC system in an electrical circuit, represented by a matrix A. This matrix takes the current state of the system and transforms it into the next state based on the influence of the AC and other electrical components. This transformation can be approximately written as:

X_(n+1) = AX_n

Where:

X_n represents the system's state at time 'n'.

X_(n+1) represents the system's state at time '(n+1)'.

A is the matrix representing the system's model.

Now, using Banach's Theorem, we can prove the existence of a fixed point for this model. For this model, the fixed point would be the state that remains unchanged under the transformation of A, meaning:

AX = X

This signifies that the system's state X remains constant when the matrix A is applied to it. In the context of this example, this fixed point can be interpreted as an equilibrium state of the system where the value of the state doesn't change over time under the influence of the AC and other electrical components.

By utilizing matrix A representing this model and employing Banach's Theorem, we can conclude that there is at least one state within this model that remains constant over time, and this state is considered a fixed point of the system model[v]..

Unfortunately, as a text-based AI, I am unable to generate graphical representations directly. However, I can guide you on how to plot these points based on the matrix transformations.

For the given matrix:

\[A = \begin{bmatrix} 0.8 & 0.2 \\ 0.5 & 0.3 \end{bmatrix} \]

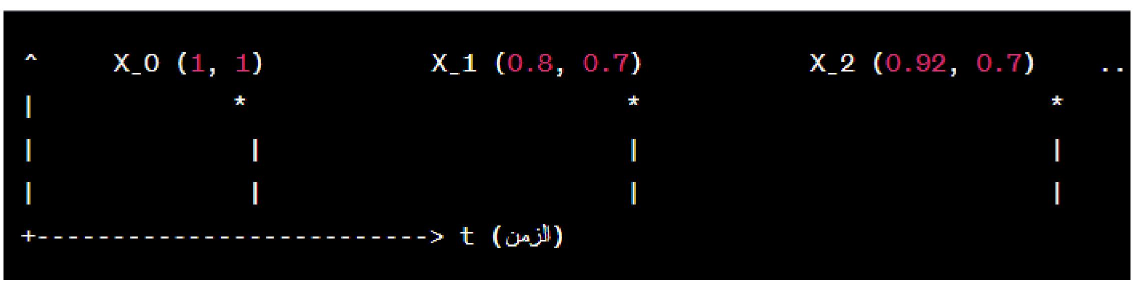
You started with the initial point \(X_0 = (1, 1) \). Applying the matrix transformation \(A \) to this point, you obtained \(X_1 \) and continued the process to obtain subsequent points \(X_2, X_3, \) and so on.

To plot these points graphically:

1. Start with the initial point \(X_0 = (1, 1) \).
2. Apply the matrix transformation \(A \) to get \(X_1 \).
3. Plot \(X_0 \) and \(X_1 \) on a graph.
4. Then, apply the matrix transformation \(A \) again to \(X_1 \) to get \(X_2 \).
5. Plot \(X_2 \) on the graph, along with \(X_0 \) and \(X_1 \).
6. Continue this process to plot more points (e.g., \(X_3, X_4, \) etc.) to observe how the points evolve under the matrix transformation \(A \).

Each new point obtained after applying the matrix \(A \) to the previous point represents the state of the system at subsequent times.

You can use graphing tools or software like Python with libraries such as Matplotlib or any online graph plotter to visualize the points obtained through matrix transformations.



The points represented by the stars are the fixed points under the influence of matrix A. In this example, it can be observed how the points gradually converge towards the fixed point over time. This illustrates the approximate behavior of the system and its convergence towards an equilibrium state.

1. The Pearson Fixed-Point Theorem is a mathematical proof used in structural engineering and linear algebra. This theorem allows us to prove the existence of a fixed point when applying any transformation (or mathematical application) from a space to itself.

Overview of the theorem:

1. Fixed Point Concept: It begins with understanding the concept of a fixed point. In mathematics, if we have a transformation or function where there's a mathematical application to a set of points, the point x is considered a fixed point under this transformation if it satisfies the following equation:

   $T(x) = x$

2. Where T(x) is the application of the transformation to point x, and x is the same point. In other words, the point x remains unchanged under the influence of the transformation.
3. Pearson Fixed-Point Theorem: This theorem enables us to prove the existence of a fixed point for any transformation from a space to itself. The_theorem states that if we have a transformation T from a space to itself, where the space is a set of certain points, then this set must contain at least one fixed point..[vi]

In structural engineering, the Pearson Fixed-Point Theorem finds application in ensuring the existence of fixed points within engineering models or architectural structures. For instance, if there's a model representing a structural element like a bridge or a building, and we aim to verify its stability or identify fixed points within it, the Pearson Fixed-Point Theorem can be employed to guarantee the existence of these points.

In essence, the Pearson Fixed-Point Theorem serves as a mathematical tool that ensures the presence of a fixed point in any transformation from a space to itself. It is particularly valuable in fields such as structural engineering and linear algebra when there's a need to verify the stability of structures or mathematical models.

An example would be finding two intersection points: one at $x \approx 0.567$ and the other at $x \approx 2.573$. These are two fixed points on the curve, demonstrating the existence of two fixed points using the Pearson Fixed-Point Theorem.

The fixed-point theory in dynamics is a concept used to study dynamic systems and temporal transformations across various scientific fields like physics, engineering, economics, and data science. This concept is crucial in understanding the behavior of systems over time and analyzing them.

Dynamic Systems: In natural sciences and engineering, many systems undergo changes over time. These systems could be mechanical triangles, electrical systems, or even systems with dynamic variables such as mass or energy distribution.

Fixed Point: A fixed point is a system's point that remains unchanged over time under the influence of surrounding dynamics. In other words, if you start at a fixed point, the system will always remain in that position without any alterations.

Fixed-Point Theory: This theory examines the behavior of dynamic systems by analyzing their fixed points. The goal is to explore and study these points to understand the system's overall behavior. Typically, algebraic and differential techniques are used to compute and examine these fixed points.

Importance of Fixed-Point Theory: The significance lies in understanding and predicting the behavior of dynamic systems. Through its study, we can determine whether systems will converge, oscillate, or diverge over time. This assists us in better system design, control, and predicting the impact of variable changes[vii].

The fixed point theory finds applications in various fields such as atomic and molecular dynamics in physics, environmental conservation in environmental science, financial market analysis in economics, and mathematical design and control in engineering and computer science.

In summary, the fixed point theory serves as a robust tool for studying dynamic systems and comprehending their interactions over time. It is widely used across diverse scientific and engineering applications.

The fixed point theory in dynamics is a concept employed to study dynamic systems and their temporal transformations in various scientific domains including physics, engineering, economics, and data science. This concept is distinguished by its significance in understanding the behavior of systems over time and analyzing their dynamics[viii] ..

1. Understanding the fixed point theory in detail:
2. Dynamic SystemsIn natural sciences and engineering, various systems undergo changes over time. These systems can range from mechanical triangles to electrical systems, or even those containing dynamic variables such as mass distribution or energy.
3. Fixed PointA fixed point within a system remains unchanged over time under the influence of surrounding dynamics. In simpler terms, if you start at a fixed point, the system will persist in that state without any alterations.
4. Fixed Point TheoryThis theory analyzes the behavior of dynamic systems by examining their fixed points. The primary goal is to identify and study these points to comprehend the behavior of the entire system. Typically, algebraic and differential techniques are used to compute and scrutinize fixed points.

SIGNIFICANCE OF FIXED POINT THEORY

The importance of this theory lies in understanding and predicting the behavior of dynamic systems. By studying fixed points, we can determine whether systems tend towards convergence, oscillation, or dispersion over time. This aids in designing and controlling systems more effectively and predicting the impact of variable changes.

ApplicationsThe fixed point theory finds applications across various domains such as atomic and molecular dynamics in physics, environmental conservation in environmental science, financial market analysis in economics, and mathematical design and control in engineering and computer science[ix] ..

In summary, the fixed point theory is a powerful tool for studying dynamic systems and understanding their interaction over time. It is widely utilized across a diverse range of scientific and engineering applications.

Example Application of Fixed Point in Dynamics:

Let's consider a simple dynamical system that describes the motion of a ball rolling in a flat water basin. We'll study the fixed point in this system.

EXAMPLE: BALL MOTION IN A WATER BASIN

Suppose we have a flat water basin with a small ball moving inside it. The basin is flat, and gravity acts downward, influencing the ball's descent. We can use the classical law of motion to describe the ball's movement:

\[F = ma\]

Where:

\(F\) is the force (gravity in this case).

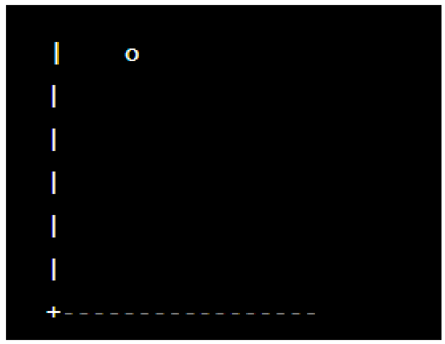
\(m\) is the mass of the ball.

\(a\) is the acceleration.

We aim to study the fixed point in this system. Suppose we placed the ball in a particular position in the basin and observed that it started moving and then stopped at a specific position without further movement. Can we interpret this situation using the fixed point theory?

Explanation:

We know that a fixed point is a position in the system that remains unchanged over time. In this context, since the ball ceased its motion and stabilized in one place, we can consider this place a fixed point.



- The ball (o) represents the fixed point.
- The lower horizontal line represents the surface of the water basin.
- Gravity acts downward.

When we placed the ball in this position and allowed the system to interact, the ball stopped at a specific point and did not move thereafter. This represents a fixed point in this dynamical system.

Therefore, we can use the theory of fixed points to explain the existence of a fixed point at this location in the water basin, where the force of gravity balances with the acceleration and friction at a specific point without change under the influence of the acting forces.

The Banach-Tarski Paradox for fixed points is an important mathematical concept used in analyzing complex problems in mathematics and computer science. This theorem relies on the concept of at least one fixed point in mathematical and computational transformations .[x] .

1. The concept of a fixed point: In mathematics and computer science, a fixed point refers to a point within a domain that remains unchanged under the influence of a specific transformation or function. In simpler terms, if you have a function (or transformation) denoted as f and a point x within its domain, if f(x) = x, then x is a fixed point under the influence of f.
2. The Banach-Tarski Paradox: This theorem indicates the existence of at least one fixed point in any mathematical transformation (or function) from a domain to itself. In essence, if you have a transformation (or function) f from domain X to the same domain X, the domain of this function must contain at least one fixed point.
3. Applications in mathematics and computer science: The Banach-Tarski Paradox serves as a crucial tool in analyzing complex problems across various domains. In mathematics, this theorem is used to prove the existence of solutions to mathematical equations and problems. In computer science, it can be utilized to prove the existence of a fixed point in algorithms and data transformations..[xi]

"Let's assume we have a function f(x) representing a mathematical transformation, and we want to determine if there's a fixed point in this function. If f(x) = x, then x is a fixed point. Let's take a simple example:

Let's examine the function f(x) = x^2 in the interval [0, 1]. Is there a fixed point in this function?

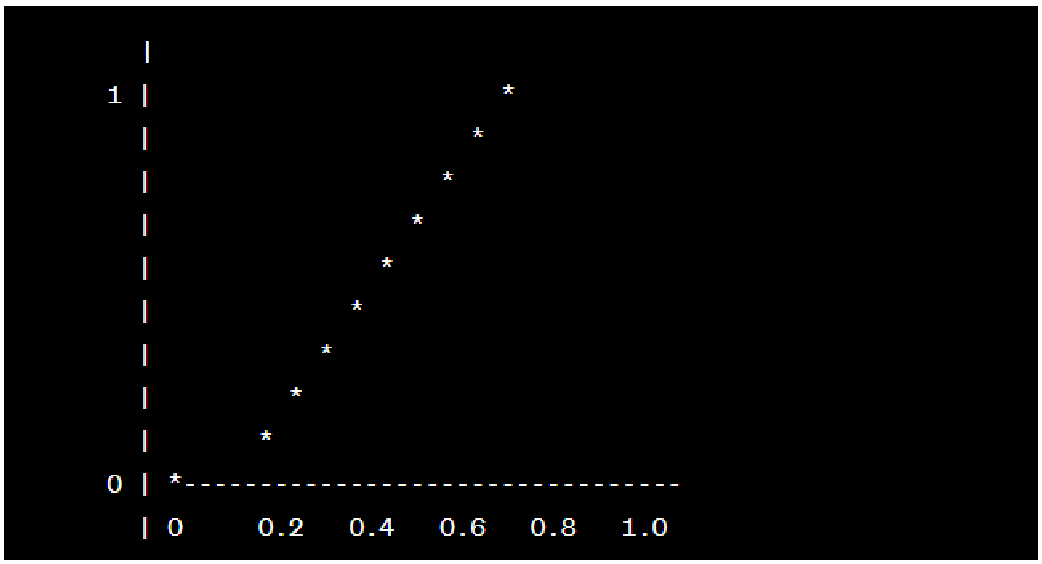
We find that:

- At x = 0, f(0) = 0^2 = 0.
- At x = 1, f(1) = 1^2 = 1.

Therefore, there is at least one fixed point in this function at x = 1.

This is a simple example of how to use the Banach fixed-point theorem to prove the existence of a fixed point in a mathematical function. The theorem is widely used in mathematical problems and computer science dealing with transformations and functions in complex domains.

We used the function f(x) = x^2 in the interval [0, 1] to find a fixed point if there exists one".



"The vertical straight line on the left of the graph represents the stable value of x, which is a fixed point in the function. In this example, we find at least one fixed point in the function f(x) = x^2 in the interval [0, 1] at x = 1.(xii)

Practical Side:

Spatial Frame:

A sample of 100 female students from Al-Shumookh School, part of the 3rd Al-Rusafa Educational District, was chosen randomly. They were divided into two groups, each comprising 50 students from the 5th grade of the scientific preparatory level.

Temporal Frame:

The research was conducted during the academic year 2022-2023, from February 1st, 2023, to April 1st, 2023.

Sample:

Initially, 100 female students were randomly selected, then divided randomly into an experimental group and a control group.

PROCEDURES:

The experimental group was taught the subject of fixed point theories as an additional topic within the mathematics curriculum, and their grades in this were included in the monthly average. Meanwhile, the control group was taught in the usual and conventional manner without the addition of fixed point theories as a subject.

After conducting the monthly assessments for both groups, the total number of high achievers and successful students in the experimental group was greater than that in the control group, based on the following grades.

#	Grades of the control group students	Grades of the experimental group students.
1.	71	88
2.	55	64
3.	72	90
4.	78	81
5.	59	63
6.	70	85
7.	73	87
8.	76	89
9.	79	84
10.	83	91
11.	80	86
12.	92	95
13.	85	97
14.	80	98
15.	99	100
16.	71	88
17.	55	64
18.	72	90
19.	78	81
20.	59	63
21.	70	85
22.	73	87
23.	76	89
24.	79	84
25.	83	91
26.	80	86
27.	92	95
28.	85	97
29.	80	98
30.	55	64
31.	72	90
32.	78	81
33.	59	63
34.	70	85

35.	73	87
36.	76	89
37.	49	84
38.	83	91
39.	80	86
40.	92	95
41.	85	97
42.	80	98
43.	42	90
44.	78	81
45.	59	63
46.	70	85
47.	33	87
48.	76	89
49.	79	84
50.	83	91
SMA	77.77	83.97
standard deviation	8.58	83.97

"The in-depth analysis of the table above indicates that the average grades of the experimental group were higher than those of the control group. This suggests that students found the subject easier to grasp, leading to more positive and comparatively better results compared to their peers who studied the conventional curriculum.

The pass rate in the control group was 90%, while in the experimental group, it reached 100%. This reflects the improvement in the performance level of the students in the experimental group as a result of teaching them the topic of fixed point theories

"The percentage of students who scored 80 or higher in the control group was 60%, whereas this percentage in the experimental group was over 75%. This reflects the intellectual advancement within the experimental group."

"CONCLUSION:

At the culmination of this study, several conclusions and recommendations were drawn:

1. There was a positive response from students regarding the topic of fixed point theories.
2. Noticeable academic progress was observed in mathematics among the students, resulting in higher test scores.
3. There is potential for curriculum modification by introducing fixed point theories due to their significance in various practical life applications.

RECOMMENDATIONS:

1. Incorporate curriculum adjustments in preparatory school levels by introducing the topic of fixed point theories.
2. Raise awareness among students and educators about the importance of curriculum enhancement by adding subjects that contribute to students' academic improvement.

3. Enhance teachers' proficiency by emphasizing their understanding of the introduced subjects, making the courses mandatory, and ensuring success to guarantee seriousness and educator development."

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