Investigation of Turbulence Models SA and Sarc for the Calculation of Weakly Swirling Currents

Murodil Erkinjonugli Madaliev, Umidjon Abdugafforugli Topvoldiev, Arabbek Baxtiyorjonugli Raxmanov, Nilufar Ulugbekqizi Qurbonova
Fergana Polytechnic Institute, Fergana, Uzbekistan

Received 25th Oct 2021, Accepted 27th Nov 2021, Online 16th Dec 2021

Annotation: The SA and SARC models of turbulence have been studied as applied to the calculation of swirling flows. Differential models of turbulent viscosity were used, taking into account the curvature of streamlines. Weakly twisted currents in a coaxial tube are considered.

Keywords: Navier-Stokes equations, SIMPLE, RANS approach, control volume method.

Introduction

Swirling flows are widely used in various technological processes, for example, to stabilize the flame, improve mixing, separation of particles, in the elements of the flow path of hydraulic power plants. Swirling currents can be accompanied by such unsteady effects as precession of the vortex core. In turn, large-scale pulsations caused by the precession of the vortex can lead to damage to structures and reduce the reliability of equipment. Thus, turbulence models are required for engineering calculations that accurately describe averaged fields and large-scale pulsations of swirling currents [1]. The scheme of the calculated area is shown in (Fig.1).

The turbulence models widely used in engineering calculations of k-ε and k-ω do not describe such flows well. To improve the modeling of turbulent swirling flows, they are trying to modify the existing RANS models (Reynolds-Averaged Navier-Stokes Equations - Reynolds-averaged Navier-Stokes equations) of turbulence. In [2], a correction to the Spalart-Allmaras (SA) model was proposed by Shar and Spalart. The new model, called SARC, was tested on a large number of swirling turbulent currents.

A system of equations averaged by Reynolds Navier-Stokes equations in a cylindrical coordinate system is used for numerical investigation of the problem [10]:

Fig.1. Diagram of the calculated area.
\[
\begin{align*}
\frac{\partial U}{\partial \xi} + \eta' \frac{\partial U}{\partial \eta} + A \frac{\partial V}{\partial \eta} &= 0, \\
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial \xi} + V \frac{\partial U}{\partial \eta} + \frac{1}{\rho} \frac{\partial P}{\partial \xi} &= \frac{1}{r \frac{\partial}{\partial r}} \left[ \nu_{\text{eff}} \frac{\partial U}{\partial \eta} \right], \\
\frac{\partial V}{\partial \xi} + U \frac{\partial V}{\partial \eta} + V \frac{\partial V}{\partial \xi} - \frac{G^2}{r^3} + \frac{1}{\rho} \frac{\partial P}{\partial \xi} &= \frac{1}{r \frac{\partial}{\partial r}} \left[ \nu_{\text{eff}} \frac{\partial V}{\partial \eta} \right] - \frac{\mu}{r^2} \nu, \\
\frac{\partial G}{\partial \xi} + U \frac{\partial G}{\partial \eta} + V \frac{\partial G}{\partial \xi} &= \frac{\partial}{r \frac{\partial}{\partial r}} \left( \nu_{\text{eff}} \frac{\partial G}{\partial \eta} \right) - \frac{\partial}{r^2 \frac{\partial}{\partial r}} \left( \frac{2 \nu_{\text{eff}}}{r^3} G \right) + \frac{2 \nu_{\text{eff}}}{r^3} G, \\
G = W \times \mathbf{r} ; \\
\nu_{\text{eff}} &= \nu + \nu',
\end{align*}
\]

Here, \( v, \nu \) are molecular and turbulent viscosity, \( U, V, W \) are dimensionless averaged velocity vectors; \( r, \xi \) are dimensionless coordinates; Initial and boundary conditions for the system of equations (1) are set in a standard way [3].

To calculate equation (1) of complex shapes, we change the coordinate system. Let's write the system (1) in the Mises variables \([8\) \( \xi, \eta \) to \(-\zeta, \eta\), where \( \zeta = \frac{z}{L} \). In the new variables, the derivatives are determined by the well-known formula:
\[
\begin{align*}
\frac{\partial}{\partial \xi} &= \frac{\partial}{\partial \zeta} + \frac{\partial}{\partial \eta} + \eta' \frac{\partial}{\partial \eta} ; \\
\frac{\partial}{\partial \eta} &= \frac{\partial}{\partial \zeta} + \frac{\partial}{\partial \eta} ; \\
A &= \frac{r - F_2(\xi)}{F_1(\xi) - F_2(\xi)} ;
\end{align*}
\]

In the new variables, the system of equations (1) takes the form
\[
\begin{align*}
\frac{\partial U}{\partial \zeta} + \eta' \frac{\partial U}{\partial \eta} + A \frac{\partial V}{\partial \eta} &= 0, \\
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial \zeta} + U \eta' \frac{\partial U}{\partial \eta} + V \frac{\partial U}{\partial \xi} + \frac{1}{\rho} \frac{\partial P}{\partial \zeta} + \eta' \frac{\partial P}{\partial \eta} &= \frac{A}{r \frac{\partial}{\partial r}} \left[ \nu_{\text{eff}} A \frac{\partial U}{\partial \eta} \right], \\
\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial \zeta} + U \eta' \frac{\partial V}{\partial \eta} + V \frac{\partial V}{\partial \xi} - \frac{G^2}{r^3} + A \frac{\partial P}{\partial \xi} &= \frac{A}{r \frac{\partial}{\partial r}} \left[ \nu_{\text{eff}} A \frac{\partial V}{\partial \eta} \right] - \frac{\nu_{\text{eff}} V}{r^2} V, \\
\frac{\partial G}{\partial t} + U \frac{\partial G}{\partial \zeta} + U \eta' \frac{\partial G}{\partial \eta} + V \frac{\partial G}{\partial \xi} &= \frac{A}{r \frac{\partial}{\partial r}} \left( \nu_{\text{eff}} A \frac{\partial G}{\partial \eta} \right) - \frac{A}{r^2 \frac{\partial}{\partial r}} \left( \frac{2 \nu_{\text{eff}}}{r^3} G \right) + \frac{2 \nu_{\text{eff}}}{r^3} G, \\

\nu_{\text{eff}} &= \nu + \nu',
\end{align*}
\]

The Reynolds-averaged Navier-Stokes equations were closed using the following turbulence models: the Spalart-Allmaras model, as well as modifications of the Spalart-Allmaras models that take into account the curvature of the current lines. In these models, Reynolds stresses are expressed in terms of strain rate tensor and turbulent viscosity.
To model the boundary conditions on the walls, the method of wall functions was used.

The Spalart-Allmaras models [2] and SARC [4,9].

This model belongs to the class of one-parameter turbulence models. Here there is only one additional equation for calculating the kinematic coefficient of vortex viscosity [11-13].

\[
\frac{\partial \bar{v}}{\partial t} + \nabla (\rho \bar{v} U) = \rho (P_v - D_v) + \frac{1}{\sigma_v} \nabla [(\nu + \nu_v) \nabla \bar{v}] + \frac{C_{k2}}{\sigma_v} \rho (\nabla \bar{v})^2 - \frac{1}{\sigma_v \rho}(\mu + \rho \bar{v}) \nabla \rho \nabla \bar{v}. \tag{3}
\]

The turbulent vortex viscosity is calculated from:

\[\nu_v = \bar{v} f_{cm} \]

The SARC model (Spalart-Allmaras for Rotation and Curvature) is the same as for the "standard" version (SA), except that the term \(P_v\) is multiplied by the rotation function \(f_{cm}\) in [4,9]. The remaining values remain the same as for the "standard" model, which are presented in [2,4]. Initial and boundary conditions. The parameters of the undisturbed flow in the entire computational domain were set as initial conditions. Non-reflective boundary conditions were applied at the outer boundary. A condition of adhesion was set on the surface of a solid. In the turbulence models SA, SARC, the value of the working variable on the body was set to zero \(\nu_v = 0\), at the input boundary \(\nu_v = 3\), the Neumann condition was set at the output boundary [14-16].

Solution method

The numerical solution of the presented system of equations was carried out in the physical variables velocity - pressure by physically splitting the velocity and pressure fields [5]. According to this method, the solution of the Reynolds equations, written in vector form, includes two stages [17-20]:

\[
\frac{\bar{U} - U^n}{\Delta t} = \nabla p^n + F(\bar{U}, U^n), \tag{4}
\]

\[
\frac{U^{n+1} - \bar{U}}{\Delta t} = -\nabla (\Delta p). \tag{5}
\]

Equation (4) is a system of equations (2) written in symbolic form and vector form. The superscript "\(\bar{U}\)" denotes an intermediate grid function for the velocity vector; \(\Delta p = p^{n+1} - p^n\) pressure correction. Multiplying equation (5) by the gradient and taking into account the solenoids of the velocity vector on the \((n+1)\)th time layer, we obtain the Poisson equation for determining the pressure correction:

\[
\nabla^2 (\Delta p) = \frac{\nabla \bar{U}}{\Delta t}. \tag{6}
\]

The solution of the stationary problem is carried out by the method of time determination, therefore the dependence (6) is written in the form of a non-stationary differential equation

\[
\frac{\partial \Delta p}{\partial t} - \nabla^2 (\Delta p) = -\frac{\nabla \bar{U}}{\Delta t}. \tag{7}
\]
where the dummy time $t_0$ is an iterative parameter. When solving equation (7) for a time step, it is possible to write $\Delta t_0 = A \Delta t$, while the value of the constant $A$ is usually less than one and is selected from the condition of rapid convergence of the numerical process. The Neumann condition is used as a boundary condition for pressure correction, which is $\bar{U}$ fulfilled if the exact value is used for the boundary $U_{i,j}^{n+1}$ [6, 7].

Thus, first, the system of equations (4) is solved by the method of determination, then equation (7) and, in accordance with (5), the velocity vector on the $(n+1)$ th time layer and pressure are determined.

$$\Delta p_{i,j}^{n+1} = p^n + \Delta p$$

Using the method of splitting the velocity and pressure fields, we obtain a system of equations (2), (3), (7), each of which is a scalar quantity transfer equation written in a conservative divergent form. The numerical solution of the transfer equation is carried out on a hybrid, staggered difference grid by the control volume method. The convective and diffusive terms of this equation are represented in finite differences using an exponential scheme [7], which provides second-order accuracy in coordinates and removes the restriction on the Reynolds grid number. In particular, for the desired variable $F$, the convective terms of the transport equation in projection, for example, on the $r$ axis, using an exponential scheme on the $(n)th$ layer in time are written as

$$U \frac{\partial \Phi}{\partial \xi} + (U \eta + V \times A) \frac{\partial \Phi}{\partial \eta} = 0.5(UU + |U|) \Phi_{i,j}^{n} - \Phi_{i,j+1}^{n} + 0.5(UU - |U|) \Phi_{i+1,j}^{n} - \Phi_{i,j}^{n} +$$

$$+ 0.5(VV + |V|) \frac{\Phi_{i,j}^{n} - \Phi_{i,j}^{n-1}}{\Delta \eta} + 0.5(VV - |V|) \frac{\Phi_{i,j+1}^{n} - \Phi_{i,j}^{n}}{\Delta \eta}.$$

Here $UU = U_{i,j}^{n}$, $VV = (U_{i,j}^{n} \eta + V_{i,j}^{n} \times A)$ and the diffusion terms of the transfer equation in projection, on the $r$ axis, using an exponential scheme on the $(n+1)$th layer in time

$$\frac{A}{r} \frac{\partial}{\partial \eta} \left[ r \nu_{eff} \frac{\partial \Phi}{\partial \eta} \right] = A \frac{\Phi_{i,j}^{n+1} (M_{i,j+1} + M_{y}) - \Phi_{i,j}^{n} (M_{i,j+1} + 2M_{i,j} + M_{i,j-1}) + \Phi_{i,j-1}^{n+1} (M_{i,j} + M_{i,j-1})}{2r \Delta \eta^2}.$$ 

Here $i, j$ correspond to the indices of the points of the difference grid at the coordinates $r, z$. The Reynolds number was used in the calculation at $Re = 10000$. The Pascal ABC program was used to calculate the problem.

**Results**

Figure 2 shows graphs comparing the calculations of SA and SARC. The figures show the profiles of the tangential $W$, axial $U$- and radial $V$-components of the velocity in the measured sections at distances from the entrance to the coaxial pipe: $z/L = 0.3$. 

---

© 2021, CAJOTAS, Central Asian Studies, All Rights Reserved

Copyright (c) 2021 Author (s). This is an open-access article distributed under the terms of Creative Commons Attribution License (CC BY). To view a copy of this license, visit https://creativecommons.org/licenses/by/4.0/
Fig. 2 shows graphs comparing the calculations of SA and SARC. The profiles of a- axial U, b-radial V and c-tangential W and velocity components in sections z/L = 0.3 are presented.

Figure 3 shows graphs comparing the calculations of SA and SARC. The figures show the profiles of the tangential W, axial U- and radial V-components of the velocity in the measured sections at distances from the entrance to the coaxial pipe: z/L = 0.6.
Fig. 3 shows graphs comparing the calculations of SA and SARC. The profiles of a- axial U, b-radial V and c-tangential W and velocity components in sections \( z/L = 0.6 \) are presented.

Figure 4 shows graphs comparing the calculations of SA and SARC. The figures show the profiles of the tangential W, axial U- and radial V-components of the velocity in the measured sections at distances from the entrance to the coaxial pipe: \( z/L = 0.9 \).

Fig. 4 shows graphs comparing the calculations of SA and SARC. The profiles of a- axial U, b-radial V and c-tangential W and velocity components in sections \( z/L = 0.9 \) are presented.

Figure 5 shows the velocity vector field in the central section of SA and SARC.
Conclusions

It can be seen from the presented figures that the numerical results of the SA and SARC models differ quite significantly. However, it is noted in [2,4] that for swirling flows, the SARC model gives closer results to experimental data. Therefore, it can be assumed that the turbulent SARC model is more suitable for describing the processes occurring inside centrifugal apparatuses.

List of literature

1. А.В. Сентябов, А.А. Гаврилов, А.А. Дектерев. Исследование моделей турбулентности для расчета закрученных течений. Теплофизика и аэромеханика, 2011, том 18, № 1 ст. 81-94.
18. Мадалиев М. Э. У. Численное моделирование течения в центробежном сепараторе на основе моделей SA и SARC //Математическое моделирование и численные методы. – 2019. – №. 2 (22).

© 2021, CAJOTAS, Central Asian Studies, All Rights Reserved


41. Рашидов, Ю. К., Исмонлов, М. М., Орзиматов, Ж. Т., Рашидов, К. Ю., & Каршиев, Ш. Ш. (2019). Повышение эффективности плоских солнечных коллекторов в системах теплоснабжения путём оптимизации их режимных параметров. In Экологическая, промышленная и энергетическая безопасность-2019 (pp. 1366-1371).

42. Рашидов, Ю. К., Исмонлов, М. М., & Исмонлов, М. М. (2019). Воздушные солнечные коллекторы: перспективы применения в условиях Узбекистана. In Экологическая, промышленная и энергетическая безопасность-2019 (pp. 1388-1390).

43. Рашидов, Ю. К., Исмонлов, М. М., Рашидов, К. Ю., & Файзие, З. Ф. (2019). Определение оптимального количества расчётных слоев многослойного водяного стратификационного аккумулятора теплоты при расчете саморегулирующегося активного элемента. In Экологическая, промышленная и энергетическая безопасность-2019 (pp. 1372-1376).


47. Усмонова, Н. А., Негматуллоев, З. Т., Нишонов, Ф. Х., & Усмонов, А. А. (2019). Модели закрученных потоков в строительстве Каркидонского водохранилища. Достижения науки и образования, (12 (53)).

