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Review of Recent Uncertainty Strategies within Optimization Techniques

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Abstract: *Despite the great progress in improvement methodologies, modernity may be a precedent for this progress. Actually, on the supply chain management scenario the decision-making becomes more challenging especially that various sources of model uncertainty are required to ensure the quality of the solution or even practical feasibility. Therefore, one of the most pressing problems today is incorporating variability in process parameters such as manufacturing time and reaction conditions. In this paper, some interactive methods are summarized that modify the actual plan obtained from the authoritative version of the system to correspond to the modifications or updated system data. Finally, the methods of dealing with problems were divided into two main approaches, the reactive approach and the preventive approach.*

Keywords: *Robust optimization, Model Predictive Control, Stochastic Programming, Fuzzy programming methods, Rolling-horizon approach.*

I. INTRODUCTION

Optimization is an important and effective area when considering the study of systems at the scheduling level, systems of physical and chemical reactions, industrial planning, site and transportation difficulties, resource allocation in engineering design and financing systems.5. It was recognized from the outset of the application of optimization to these challenges that natural and technical system analyzers are virtually always confronted with uncertainty [1,2,3,4,5,6,7,8,9]. This review's main goal is to give a quick understanding of optimization under uncertainty. Since the seminal works of Beale (1955), Bellman (1957), Bellman and Zadeh (1970), Charnes and Cooper (1959), Dantzig (1955), and Tintner (1955), both the theory and techniques of optimization under uncertainty have seen substantial development (1955). It marked the beginning of the launch with works by Bertsekas and Tsitsiklis (1996), Birge and Louveaux (1997), Kall and Wallace (1994), Prékopa (1995), and Zimmermann (1991), as well as the very extensive Stochastic Programming Community Home Page (2003), [10,11,12,13,14,15]. In order to provide a brief overview, the methods of dealing with problems have been divided into two main approaches, the reactive approach and the preventive approach, which will be dealt with in more detail, see (Fig. 1):

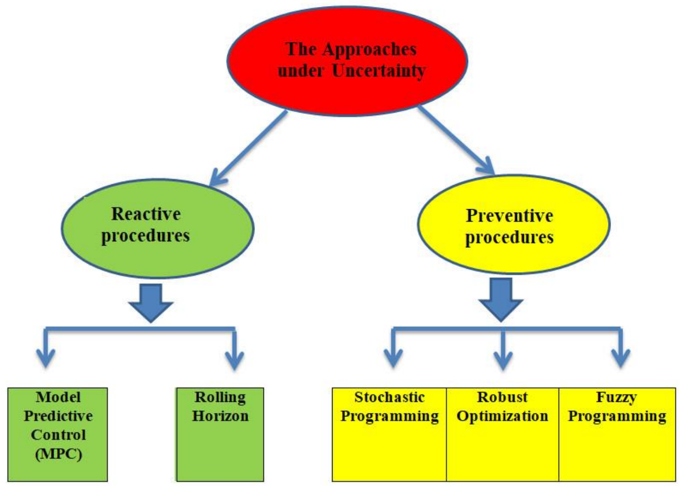

Fig.1. Classification under uncertainty.

II. REACTIVE PROCEDURE

A. Model Predictive Control

Model predictive control (MPC) was first established in the late 1970s. In fact, this model plays a fundamental and important role nowadays. In addition to being an academic method , it is considered an industrial method that mediates many multivariate processes, especially since it makes extensive use with advanced process control (Pistikopoulos, 2009). Due to its ability to manipulate and manipulate multivariate interactive processes at scale, generate predictions, increase performance, and respect constraints, Predictive Model Control (MPC) has a wide area for more applications. According to the degree of technological advancement, the evolution of this plan can be split into three stages as shown in (Fig.2) depicts the MPC theoretical premise, [16,17,18] .

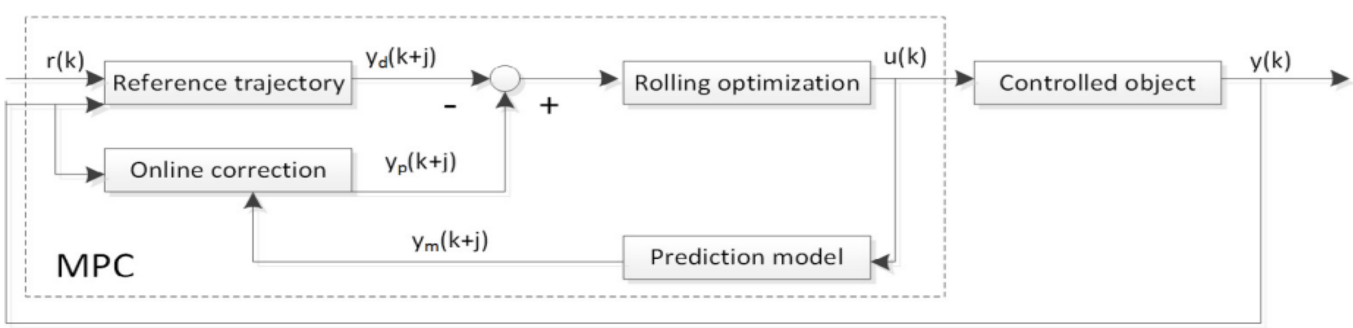

Fig.2. MPC theoretical premise.

As shown, the starting point will be by input (r (k)) there will be three outputs (initial output, output after optimization, and output after correction is y(k)). The waterside equipment operating schedule and the set points sent to the atmospheric regulatory controllers can be calculated via the Building Automation System (BAS) and this technique is done by solving the optimization problem at each time step, as shown in (Fig. 3).

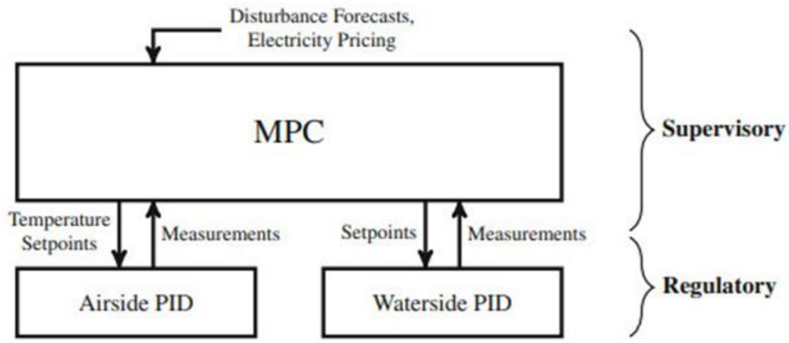

Fig.3. An example of a predictive control model in an air side building automation System (BAS) within PID regulatory controllers.

for example, building models can be used to calculate the loads. Also, waterside system models can be used to establish the associated power consumption, the relevant set point dynamics can be incorporated into power consumption and regulatory controller models. Since the purpose of MPC improvement is to reduce building energy costs, it is necessary that the application of the economic cost function take the starting point to reach the primary objective that results in an economic MPC framework. These and other studies show how naturally occurring load shifting as a result of MPC optimization can result in significant cost reductions. Despite the fact that these studies have demonstrated that MPC can provide significant benefits, MPC-based systems have yet to be widely used. MPC has the main advantage of being model dependent, which means that output predictions are computed using a process model. This shows that the model can account for the limits of the state and control variables. As a result, the accuracy of the process model is critical to the success of this methodology. Finally, the simplest model should be capable of making correct predictions in a reasonable length of time [19].

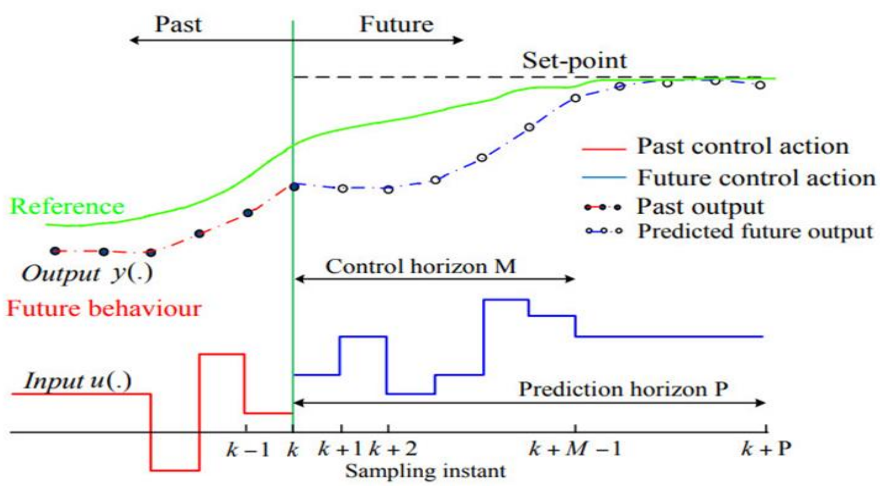

Fig.4. Model Predictive Control in general case.

Model Predictive Control Formulation

MPC is almost typically formulated in the state space in the research literature. Let the linear discrete-time difference equations define the plant model Σ to be controlled.

$$\Sigma: \begin{cases} s(a+1) & = Ws(a) + Pu(a),\ s(0) = s_0 \\ y(a) & = Cs(a) \end{cases}$$

where $s(a) \in \mathbb{R}^n, u(a) \in \mathbb{R}^m, y(a) \in \mathbb{R}^p$ indicate (the state, control information, and result) respectively. consider $s(a+k, s(a), \Sigma)$ or in other form $s(a+k \mid a)$ mean the expectation got by repeating model (1) k times from the present status s(a).

The solution to the open-loop optimization problem shown below is commonly used to implement a receding horizon:

$$\mathbf{U} \triangleq \{u(a+k \mid a)\}_{k=a}^{a+N_m-1} \quad J(\mathbf{U}, s(a), N_p, N_m) = s^A(N_p) P_0 s(N_p) + \sum_{k=0}^{N_p-1} s'(a+k \mid a) Q s(a+k \mid a) + \sum_{k=0}^{N_m-1} u'(a+k \mid a) R u(a+k \mid a)$$

subject to

$$\begin{aligned} F_1 u(a+k \mid a) &\le G_1 \\ E_2 s(a+k \mid a) + F_2 u(a+k \mid a) &\le G_2 \end{aligned}$$

And

"stability constraints" (2c)

as shown in (Fig. 4), N_p indicates the forecast or output horizon length, N_m indicates the control or input horizon's length $(N_m \le N_p)$. When $N_p = \infty$, as the endless horizon problem, and then when as a finite horizon problem where N_p is finite problem. To make the problem intelligible, we suppose that the polyhedron $\{(s,u): F_1 u \le G_1, E_2 s + F_2 u \le G_2\}$ includes the origin $(s = 0, u = 0)$. Therefore, to achieve closed-loop stability, the constraints (2c) are introduced into the optimization problem [20,21].

B. Rolling-horizon approach

The rolling horizon technique is an interactive scheduling strategy that approaches and resolves deterministic problems repeatedly by advancing the optimization horizon with each iteration. Actually, this technique does this by assuming that the system's status is updated as soon as the different uncertain or insufficiently precise parameters are known, and that the optimal schedule for the new situation and optimization horizon can be determined [22,23].

Strategy

The RH strategy's primary concept is to divide work into various task sets with specific overlaps based on the arrival order, and the division can be changed in real time as the scheduling period progresses. The rolling horizon refers to the fact that each scheduling will decide and only allocate its job set.

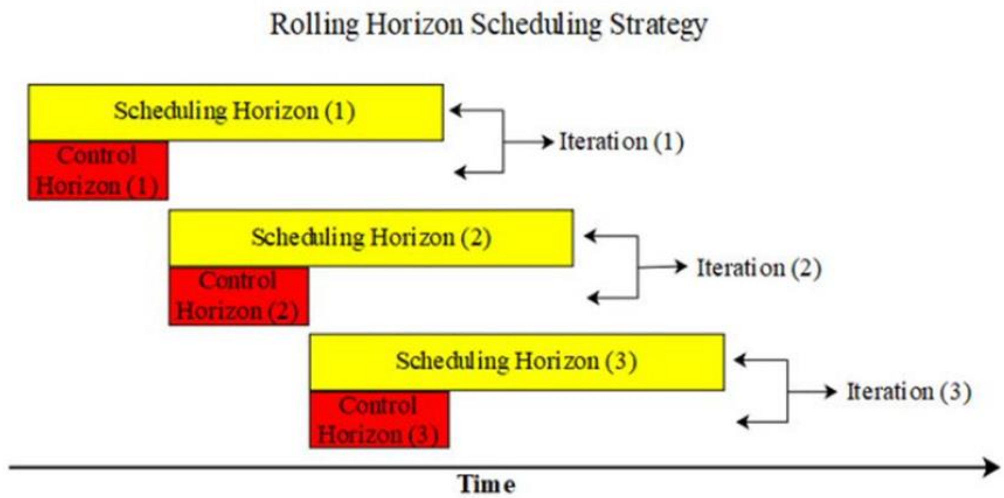

Fig.5. A rolling horizon framework is used for reactive scheduling.

This technique takes into account a prediction horizon in which all uncertain parameters associated with this time horizon are considered to be known with certainty, and a control horizon in which optimization choices for the prediction horizon are implemented. Several scheduling problems involving uncertainty have been solved using the rolling-horizon method. The rolling horizon strategy used in this study avoids infeasible situations by allowing backlogs to build up if future demand exceeds what the model could identify in the sub-problems earlier, see (Fig.5). Finally, from the point of view of the fact that seeks that backlogs are penalized in the objective function, worse solutions may emerge. The rolling time horizon strategy, on the other hand, outperformed the whole model in finite time in the base scenario provided here [24,25,26,27,28].

III. PREVENTIVE PROCEDURE

A. Stochastic Programming approach

Following foundational breakthroughs in linear and nonlinear programming, the field of stochastic programming was founded in the mid-nineteenth century. While it was rapidly apparent that the inclusion of uncertainty in optimization models necessitates novel problem formulations, it took many years to develop and analyze the basic stochastic programming models. Today, stochastic programming theory provides a number of methods to deal with the inclusion of random data in optimization problems, including chance-constrained models, two- and multi-stage models, and risk-measure models. Almost every year, new problem formulations emerge, and this diversity is one of the field's strengths [29,30,31]. Stochastic programming is based on complex mathematical tools such as no smooth calculus, abstract optimization, probability theory, and statistical approaches, and can be fairly complicated, starting with sophisticated modeling. It is important to distinguish between two sets of choice variables in a broad stochastic optimization problem:

1- First and foremost, stage one decisions are those that must be made before any ambiguous parameter is exposed. They're also referred to as "here and now" choices.
2- After part or all of the unknown data is exposed, resources are determined. The second and subsequent

stages of decision-making are sometimes known as "wait and see" decisions.

The two-stage stochastic formulation is the most extensively used and simplest stochastic program. The vector a represents the first stage decisions, the vector b represents the second stage decisions, and the vector c represents the uncertain parameters. The second-stage choices b are influenced by the first-stage decisions x and unanticipated occurrences. The Q function is then added to simplify the representation of the problem .

$$Q(a,c) = \min_x f_2(b,c)$$
subject to
$$h_2(a,b,c) = 0$$
$$g_2(a,b,c) \leq 0$$
$$b \in Y \subset \mathbb{R}^{n2}$$
$$f_2: \mathbb{R}^{n2} \to \mathbb{R}$$
$$h_2: \mathbb{R}^{n2} \to \mathbb{R}^{l2}$$
$$g_2: \mathbb{R}^{n2} \to \mathbb{R}^{m2}$$

Thus, Q is a mathematical program that minimizes the value of the unknown coefficient c in the second stage variable. Also, Q takes into account all equations that involve recourse decisions b. The expression below defines the expected recourse function Q:

$$\mathrm{Q(a) = E_c[Q(a,c)]}$$

Approximations to the continuous distribution behavior can be derived by generating a discrete number of possibilities. The continuous probability functions of a stochastic program can be approximated to discrete functions using sampling techniques in this scenario [32,33,34,35,36,37]. Finally, A scenario tree can be used to show the combination of numerous scenarios that can occur (Fig. 6).

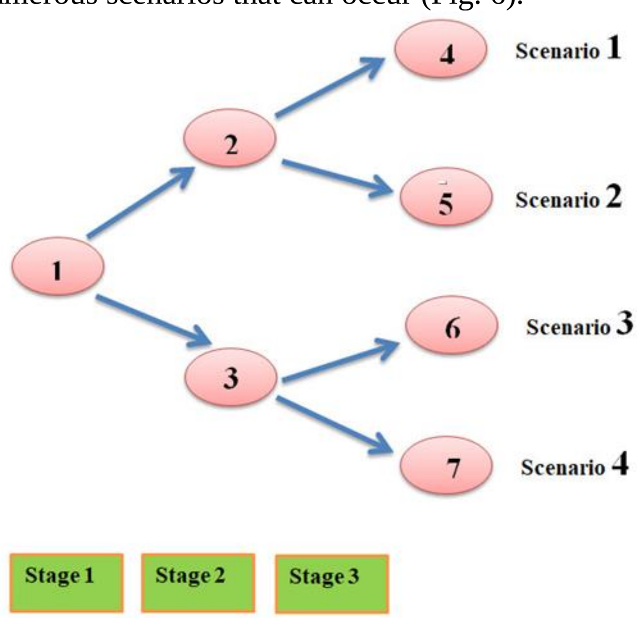

Fig.6. An example of stochastic programming (Scenario tree).

B. Robust optimization approach

Robust optimization is a modeling methodology that uses a deterministic approach. In fact, an optimal solution is required for any implementation of uncertain transactions within the specified uncertainty sets according to the modeling technique. Therefore, this approach is comparable to the resort model of stochastic programming in that the part of the parameters that are random variables, and here the term (scenarios) is used as an expression of the alternative achievements of the penalty function in the target ,[38,39,40].

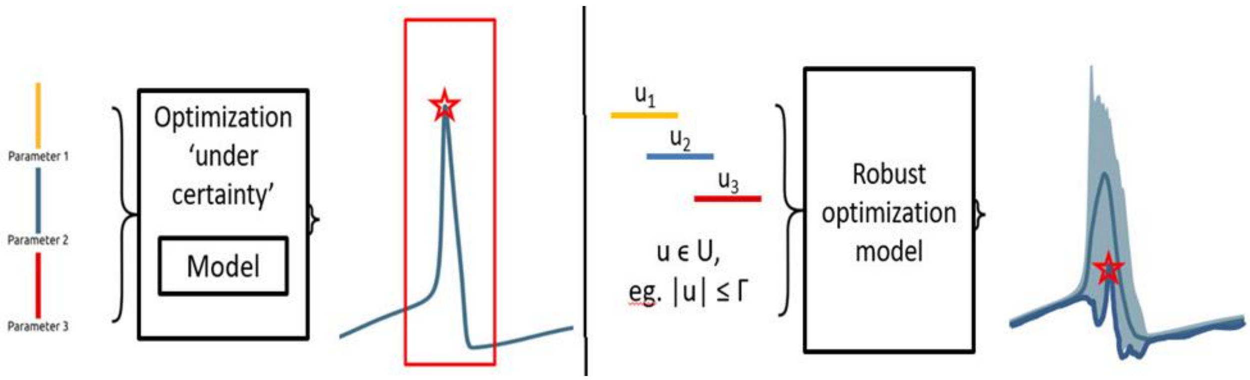

Fig.7. Comparison between Optimization models 'under certainty' and Stochastic optimization.

C. Fuzzy programming methods

The phrase "fuzzy" describes anything that lacks precision or clarity. In the real world, we often meet circumstances in which it is impossible to discern whether a condition is true or false; fuzzy logic gives a crucial degree of mental flexibility. In the Boolean system, for instance, the number 1.0 denotes absolute truth and 0.0 represents absolute falseness. The major contrast between stochastic programming and robust optimization and fuzzy optimization methodologies is how uncertainty is handled. In this proactive method, random parameters are represented as fuzzy integers, while limits are regarded as fuzzy sets. The constraint degree fulfillment is determined The constraint degree fulfillment is determined by the constraint's membership function, and certain constraint breaches may be accepted, and certain constraint breaches may be accepted. Objective functions are treated as constraints in fuzzy mathematical programming, with the lower and higher bounds influencing the decision-expectations. There are additional ways to explain uncertainty than fuzzy logic and probability is to consider that both fuzzy logic and probability theory are capable of expressing subjective belief. Fuzzy set theory uses the idea of fuzzy set membership, which implies (how much of a variable is contained in a set) as in shown in (Fig.8). In probability theory, the idea of subjective probability (how likely do I perceive a variable to be in a set?) is used ,[41,42,43,44,45,46,47,48,49].

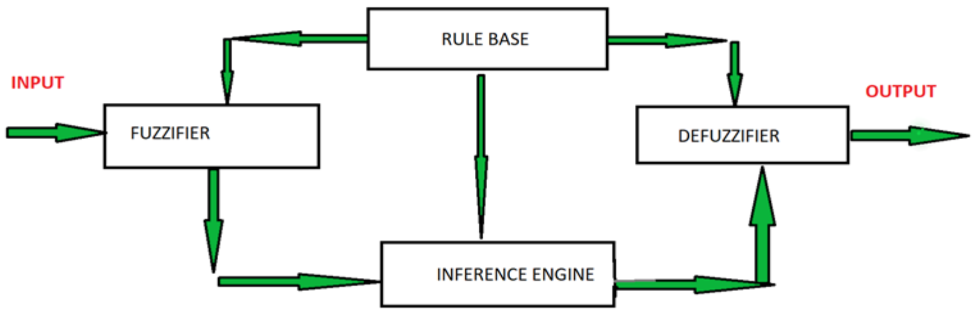

Fig.8. Fuzzy Logic Architecture.

CONCLUTION

A brief summary of recent uncertainty strategies is reported under Optimization Techniques. In fact, the methods of dealing with problems have been divided into two main approaches, the reactive approach and the preventive approach. Also, each approach is briefly introduced according to the optimization approach. Moreover, the interactive method included two main methods, while the preventive method included three methods. Finally, the preventive approach relied on the methods of dealing according to the updated variables on the basis of the reactive approach, which ultimately aims to meet the objective function and constraints of optimization problems.

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