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K-functional for Neural Networks and r-th Smoothness Moduli

Sarah A. AL-Ameedee[1], Ahmed J. Obaid[2], Zeyad Kadhim Jabber[3], Fouad Abdualameer Salman[4], Awatif Ibrahim Mohammed Alhaddad[5]

[1] Al-Mustaqbal University college, Department of Computer Technical, Iraq.
[2] Faculty of Computer Science and Mathematics, University of Kufa, Iraq.
[3, 4, 5] College of Business Administration, Al-Bayan University, Baghdad, Iraq.

[1]sarah.abdulridha@uomus.edu.iq , [2]ahmedj.aljanaby@uokufa.edu.iq, [3]zeyad.kadhim@albayan.edu.iq, [4]fouad.a@albayan.edu.iq, [5]awatif.ibrahim@albayan.edu.iq

Abstract: We introduce a relationship between the r-th modulus of smoothness and the K-functional of J-peetre in this article.

Keywords: smoothness, functional, modulus, network.

1- INTRODUCTION

K-functional peetre's is a useful tool for describing the smoothness of functions, particularly those defined on and belonging to. For functions defined on and belonging to this article, we will prove an equivalent between it and the r-th order modulus of smoothness.

Definition 1.1 [3]

A k-function can be defined as follows:

$$K_2(f,t^2) = \inf_{D^{|m|}g \in Q.W.loc}\left\{\|f-g\| + t^2 \, sup_{|m|=2}\left\|D^{|m|}g\right\|\right\}$$

Where $g \in Q.W.loc$ means that g is $|m|$ times differentiable and $D^{|m|}g$ is continuous in the finite set.

Definition 1.2 [2]

Let $g_s(x)$

$$g_s(x) = -\sum_{k=1}^{m}(-1)^{m-k}\binom{m}{k}\int_0^1 \dots \int_0^1 f(x+k).\sum_{h=1}^{n}\sum_{t=1}^{m} sc_{ij}\, e_i)\prod_{h=1}^{n}\prod_{t=1}^{m} d\,\partial_{ht}$$

Definition 1.3 [2]

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$$D^{\beta} g_s(x) = -s^{-m}\sum_{k=1}^{m}(-1)^{m-k}\binom{r}{k}\int_0^1 \dots \int_0^1 \mathrm{E}(\mathrm{ks}\sum_{\mathrm{h}=1}^{\mathrm{n}}\sum_{\mathrm{t}=1}^{\mathrm{r}-\beta_{\mathrm{i}}} \epsilon_{\mathrm{ij}}\mathrm{e}_{\mathrm{i}}).\prod_{h=1}^{n}\Delta^{\beta_i}(kse_i)f(x)\prod_{h=1}^{n}\prod_{t=1}^{m-\beta_i} d\partial_{ij}.$$

2- Auxiliary Results

Lemma 2.1[7]

If $f \in \mathrm{L_p}(\mathrm{R^d})$, then

$\tau_{\mathrm{r}}(f,t) \le o(p)\|f\|_p$

Lemma 2.2 [3]

If $f \in L_p^{(r)}(R^d)$ **,** we have

$\tau_r(f,t) \le c(p)t^k\tau_{r-k}(D^k f,t)_p$.

Lemma 2.3 [1]

If $f \in L_p(R^d)$we have

$\tau_r(f,ct)_{\mathrm{p}} \le o(p)\tau_r(f,t)_p$.

Where $c \in R$.

Lemma 2.4 [4]

If $f \in L_p(R^d)$and if $s_1 < s_2$ then

$\tau_r(f,s_1)_p \le \tau_r(f,s_2)_p$

3- The Main Result

Theorem 2.1

If $f \in L_p(R^d)$

$$K_{\mathrm{r}}(f,t^r)_p = \inf_{\mathrm{D}^{|\mathrm{m}|}\mathrm{g}\in\mathrm{L_p^{(m)}}}\|\mathrm{f}-\mathrm{g}\|_{\mathrm{p}} + t^r{}_{|m|=r}\left\|D^{(m)}g\right\|_p$$

$$o(p)K_r(f,t)_p^r \le \tau_r(f,t)_p \le o(p)K_r(f,t^r)_p$$

Proof:

If $f \in L_p(R^d)$,

then

$\tau_r(f,t)_p \le o(p)K_r(f,t^r)_p$

By Lemma 3.1 and Lemma 3.2 we get

$\tau_r(f,t) = \tau_r(f-g+g,t)_p$

$\le o(p)\tau_r(f-g,t)_p + \tau_r(g,t)_p$

$\le o(p)\left(\|f-g\|_p + t^r\left\|D^{(m)}g\right\|_p\right)$

If $f \in L_p(R)^d$, then

$K_r(f,t)_p \le c(p)\tau_r(f,t)$

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suppose e_j be the unit vectors in R^d, $0 < s \leq 1$,

Putting $f(l+b) = E(l)f(b)$ in Definition 2.3 , by Lemma 3.4 we obtain

$$\|f - g_s\|_p = \left\| \int_0^1 \dots \int_0^1 \Delta^r \left(s \sum_{h=1}^{n} \sum_{t=1}^{r} \theta_{ij}\, e_i \right) f \prod_{h=1}^{n} \prod_{t=1}^{r} d\,\theta_{ij} \right\|_p$$

$$\leq c(p)\tau_r(f,(n+r)s)$$

by Definition 2.3, and Lemma 3.4

$$\left\|D^\beta g_s\right\|_p \leq s^{-r} \sum_{k=1}^{m} \binom{m}{k} \int_0^1 \dots \int_0^1 \left\| E\left(ks \sum_{h=1}^{n} \sum_{t=1}^{m-\beta_i} \theta_{ij} e_i \right) \prod_{h=1}^{n} \Delta^{\beta_i}(kse_i) f \right\|_p$$

$$. \prod_{h=1}^{n} \prod_{t=1}^{r-\beta_i} d\theta_{ij}.$$

$$\leq o(p).s^{-r}\tau_r(f,(n+m)s)_p$$

let $s = \frac{1}{(n+r)^2}$ $0 < s < t$

by Lemma 3.3.3 and Lemma 3.3.4 to get

$$\left\|D^\beta g_s\right\|_p \leq o(p)t^{-r}\tau_r(f,t)_p$$

$$\|f - g_s\|_p \leq c(p)\tau_r(f,t)_p. \blacksquare$$

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