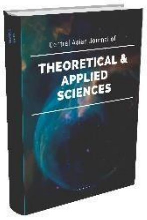

CENTRAL ASIAN JOURNAL OF THEORETICAL AND APPLIED SCIENCES

Volume: 03 Issue: 05 | May 2022 ISSN: 2660-5317

Inverse Estimation by Spherical Neural Networks

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Abstract: we discuss an inverse estimation method for approximating functions in L p. Spherical neural networks are used to compute spaces for $p < 1$.

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Keywords: spherical, functional, neural, network.

1- INTRODUCTION

Suppose $E^m = \{\alpha = (\alpha_1, \alpha_2, \dots, \alpha_m) : \alpha_i = \pm 1 \; for \; each \; l = 1, \dots, m\}. \; m \in \mathbb{N}$.

We denoted algebraic polynomial with variables $\sigma_k \in R$ by $P_{ij}(\sigma_k)$ for each $k = 1,2,\dots,p$, $l = 1,2,\dots,m, w = 1,\dots,q, P_l(b_d,\sigma_k) = \sum_{d=1}^{q} b_w P_{lw}(\sigma_k)$.

And $\mathcal{P}_{m,s,p,q} = \{(\pi_1(b_d,\sigma_k),\dots,\pi_m(b_d,\sigma_k))\}$,

$H_{2r}^2(S^d)$ the class of function f for which $\Delta^r f := \Delta^{r-1}(\Delta f), r = 2,3,\dots$

W_{2r}^2 the Sobolev space subset of $H_{2r}^2(S^d)$ and $\|\Delta^r f\|_2 \le 1$, $f \in H_{2r}^2(S^d)$

And $\Delta H_k = -\lambda_k H_k$, $H_k \in H_k^d$, $\lambda_k := k(k+d-1), k = 0,1,\dots$.

$\prod_s^d$ class of spherical harmonics with degree k,

H_k^d class of all SPs with degree $k \le s$

Where $\prod_s^d$ is $\sum_{k=0}^{s} d_k^d = d_s^{d+1} \sim s^d$.

$\prod_s^d = \bigoplus_{k=0}^{s} H_k^d$ and $L^2(S^d) = closure\{\bigoplus_k H_k^d\}$

If basis $\{Y_{l,w} : l = 1,\dots,d_k^d\}$ for H_k^d then $\{Y_{l,w} : l = 0,1,\dots,w = 1,\dots,d_k^d\}$ is basis for $L^2(S^d)$

$$d_k^d := dim\, H_k^d = \begin{cases} \frac{2k+d-1}{k+d-1}\binom{k+d-1}{k}, & k \geq 1; \\ 1, & k = 0, \end{cases}$$

We define Laplace-Beltrami operator Δ [1] by

$$\Delta f := \sum_{i=1}^{d+1} \frac{\partial^2 h(x)}{\partial x_i^2}\Big|_{|x|:=(x_1^2+x_2^2+\cdots+x_{d+1}^2)^{\frac{1}{2}}=1}, h(x) = f(\frac{x}{|x|})$$

Definition 1.2 [2]

Let I be an interval in $\mathbb{R}$, $f: I \to \mathbb{R}$ is absolutely continuous function on I if for every $\epsilon > 0$, there is $\delta > 0$ for a finite sequence of (x_k, y_k) of I where

$\sum_k (y_k - x_k) < \delta$ then $\sum_k |f(y_k) - f(x_k)| < \epsilon$

Lemma 1.1 [3]

Suppose m, p, q, s be integers such that $u + v \leq \frac{m}{2}$

$$u \log_2(4s) + (u+2)\log_2(u+v+1) + (u+v)\log_2(\frac{2em}{u+v}) \leq \frac{m}{4}$$

Then there is an absolute constant $C \geq 0$ and a vector $\epsilon \in E^m$.

$$d(l^p) \geq d_2(\epsilon, P_m, s, p, q, l^2) \geq \frac{C}{n^2}$$

Where $l^2 = \{x = (x_1, x_2, \dots): x_i \in \mathbb{R}\, \forall i, \sum_{i=1}^{\infty}|x_i|^2 < \infty\}$

Lemma 1.2 [4]

If $\pi_i \in \mathbb{R}$ and $0 < c < b$ then

$$(\sum_{n=1}^{\infty}|\pi_n|^b)^{\frac{1}{b}} \leq (\sum_{n=1}^{\infty}|\pi_n|^b)^{\frac{1}{c}}$$

Lemma 1.3 [2]

Suppose $r > 0, d \geq 2$, then for any $\alpha \in L^2(\mathbb{R})$ there exists a constant C depending only on d such that

$$d_e(\epsilon, P_m, s, p, q, l^e) \geq c(e)^{\frac{1}{n^2}}$$

Lemma 1.4 [3]

If $h_j (j = 1, \dots, d_s^d)$ are univariate polynomials , $\epsilon = (\epsilon_1, \dots, \epsilon_m)$, $\epsilon_i = \pm 1$, $\sum_{j=0}^{s}\sum_{i=1}^{d_j^d}\left|\in(j,i) - \langle g, Y_{j,i}\rangle\right|^2 \geq cS^{d/2}$.

2- The Main Result

Theorem 2.1

If $r \geq 0, d \geq 1$ then for any $\vartheta \in L_p(\mathbb{R})$ there exists a constant $B(d)$ satisfy $d_p(W_p^2, \vartheta_{\theta,n}, L_p(s^d) \geq B(p)n$

Where $W_p^2 = \{f: f, \Delta f \in L_p\, , \acute{f}$ is absolutely continuous }

Proof

Suppose $n, s \in \mathbb{N}, \mathrm{m} = \mathrm{d}_s^{d+1}$, then there exists $B \leq 2$ such that $d_s^{d+1} = Bs^d$, suppose $SP_{s,d=}\{F(x) = \sum_{j=0}^{s} \sum_{i=1}^{d_j^d} \epsilon_{w,l} Y_{w,l}(x)$

Where $\{\epsilon_{w,l}: w = 0, \dots, s, l = 1, \dots, d_s^d\} \subset E^d$

By definition of $Y_{w,l}$ we obtain $SP_{s,d} \subset \prod_s^d$

Now we shall prove $F \in L_p, \|\Delta^r F\|_p \in L_p$, F is an absolutely continuous

$$\|F(x)\|_p = \left\|\sum_{j=0}^{s} \sum_{i=1}^{d_j^d} \epsilon_{w,l} Y_{w,l}(x)\right\|_p \leq B(p,s,d) \left\|Y_{w,l}\right\|_p \tag{2.1}$$

If $Y_{w,l} \in L_p$, then $\|F\|_p < \infty$ and $F \in L_p$.

We have by Bernstein inequality that

$\|\Delta^r F\|_p \leq B(p) s^{2r} \|F\|_p$

By (2.1) we obtain

$\|\Delta^r F\|_p < \infty$ and $\Delta^r F \in L_p$ which implies

$F^\star(x) = CS^{-2r} m^{-\frac{1}{2}} F(x) \in W_{2r}^p$

Now we shall estimate $dist\left(SP_{s,d}, \vartheta_{\vartheta,n}, L_p\right) = sup_{F \in SP_{s,d}} \inf_{h \in \vartheta_{\vartheta,n}} \|F - h\|_p$

We have $F(x) = \sum_{j=0}^{s} \sum_{i=1}^{d_j^d} \epsilon_{w,l} Y_{w,l}(x) \in SP_{s,d}$

And we have $h(x) = \sum_{k=1}^{n} c_k \vartheta(\langle w_k, x\rangle + b_k)$

$w_k \in \mathbb{R}^{d+1}, b_k, c_k \in \mathbb{R} \in \vartheta_{\vartheta,n}$

Then

$$h(x) = \sum_{k=1}^{n} c_k \vartheta(\langle a_k, x_k, x\rangle + b_k)$$

$h(x) = \sum_{k=1}^{n} c_k \vartheta(a_k \langle x_k, x\rangle + b_k)$,$a_k b_k, c_k \in \mathbb{R}, x_k \in S^d$

It is clear that $g \in L_p(S^d)$

Because $\vartheta \in L_p(\mathbb{R}^d)$ then

$$\|F - h\|_p^p = \left\|\sum_{j=0}^{s} \sum_{i=1}^{d_j^d} \epsilon_{w,l} Y_{w,l}(x) - h(x)\right\|_p^p =$$

$$\left\|\sum_{j=0}^{s} \sum_{i=1}^{d_j^d} \epsilon_{w,l} Y_{w,l}(x) - \sum_{j=0}^{s} \sum_{i=1}^{d_j^d} \langle h, Y_{w,l}(x)\rangle Y_{j,i}(x) - \sum_{j=s+1}^{\infty} \sum_{i=1}^{d_j^d} \langle h, Y_{w,l}(x)\rangle Y_{w,l}(x)\right\|_p^p$$

$$\geq c(p) \left\|\sum_{w=0}^{s} \sum_{l=1}^{d_w^d} \in (w,l) - \langle h, Y_{w,l}\rangle - \sum_{j=s+1}^{\infty} \sum_{l=1}^{d_w^d} \langle h, Y_{w,l}\rangle\right\|_2^2 \tag{2.2}$$

Using Parseval identity we get

$$\geq c(p)\sum_{w=0}^{s}\sum_{l=1}^{d_w^d}\left|\in(w,l)-\langle h,Y_{w,l}\rangle\right|^2+\sum_{w=s+1}^{\infty}\sum_{l=1}^{d_w^d}\left|\langle h,Y_{w,l}\rangle\right|^2$$

$$\geq c(p)\sum_{w=0}^{s}\sum_{l=1}^{d_w^d}\left|\in(w,l)-\langle h,Y_{w,l}\rangle\right|^2$$

Then

$$d_p(W_{2r}^2,\vartheta_{\vartheta,n},L_p(S^d))\geq$$

$$d_p(B(p)s^{-2r}m^{-\frac{1}{2}}F_{n,d},\vartheta_{\vartheta,n},L_p(S^d))$$

$$\geq B(p)s^{-2r-\frac{d}{2}}d_p(F_{n,d},\vartheta_{\vartheta,n},L_p(S^d))$$

$$\geq B(p)s^{-2r-\frac{d}{2}+\frac{d}{2}}=c(p)s^{-2r}\sim n^{-\frac{2r}{d-1}} \quad \blacksquare$$

Theorem 2.2

if $n=cs^{d-1}$, then there exist an analytic, increasing function $\vartheta\in L_p(S^d)$ that is satisfied

$$d_p(W_p^2,\vartheta_{\emptyset,n},L_p(S^d))\sim d_p(W_p^2,\Pi_s^d,L_p^2(S^d)\sim n^{-\frac{2r}{d-1}}$$

Proof

Directly by Theorem 2.1 ■

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