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Simplification of differential equations of elastic viscous fluid motion in pipes and ducts

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Abstract: *A completely linear form of the system of equations representing the flow of elastic viscous fluid in flat channels and circular cylindrical tubes was obtained, and it was possible to obtain an analytical solution for small amplitude oscillations.*

Keywords: *Dynamic viscosity coefficient, oscillation frequency in pulsating current, non-Newtonian, Reynolds number.*

To simplify the differential equations of elastic viscous fluid motion in pipes and channels, we assume that the flow is symmetrical along the longitudinal axis, that the channel length L is much larger than the channel width h, i.e., $\frac{h}{L}=\frac{R}{L}<<1$.The ratio of the characteristic transverse velocity to the characteristic longitudinal velocity is called an infinitely small quantity, i.e., $\frac{V}{U}<<1$. These conditions are always fulfilled in the slow motion of the fluid, i.e. at small values of the Reynolds number. Given these assumptions, it is possible to simplify the system of equations describing the flow of non-Newtonian fluids in pipes. To do this, it is expedient to make the terms entering the systems of equations dimensionless using the following substitutions:

$$
\begin{aligned}
&t=\lambda t_1, && y=hy_1,\\
&x=Lx_1, && u=Uu_1,\\
&\vartheta=V\vartheta_1, && p=\frac{\eta UL}{h^2}p_1,\\
&\tau_{ij}=\eta\frac{U}{h}\tau_{1,ij}, && p_k=\frac{\eta}{\lambda}p_{1,k},\\
&\eta_k=\eta\eta_{1,k}, && \lambda_k=\lambda\lambda_{1,k}
\end{aligned}
\qquad (1)
$$

Here, t – time; U –longitudinal speed; V –transverse speed; $2h$ –channel cross section length; L -channel length; η – dynamic viscosity coefficient.

By substituting these substitutions into the system of equations and the equation of continuity, taking into account the symmetry of the current along the transverse axis, we obtain its appearance in the Cartesian coordinate system without taking into account the external mass forces:

$$\begin{cases} \dfrac{1}{EL}\dfrac{\partial u_1}{\partial t_1} + \mathrm{Re}\,\delta\left(u_1\dfrac{\partial u_1}{\partial x_1} + \vartheta_1\dfrac{\partial u_1}{\partial y_1}\right) = -\dfrac{\partial p_1}{\partial x_1} + \left(\dfrac{\partial \tau_{1,11}}{\partial x_1}\delta + \dfrac{\partial \tau_{1,21}}{\partial y_1}\right), \\ \dfrac{1}{EL}\delta^2\dfrac{\partial \vartheta_1}{\partial t_1} + \mathrm{Re}\,\delta^3\left(u_1\dfrac{\partial \vartheta_1}{\partial x_1} + \vartheta_1\dfrac{\partial \vartheta_1}{\partial y_1}\right) = -\dfrac{\partial p_1}{\partial y_1} + \delta^2\left(\dfrac{\partial \tau_{1,12}}{\partial x_1}\delta + \dfrac{\partial \tau_{1,22}}{\partial y_1}\right), \\ \dfrac{\partial u_1}{\partial x_1} + \dfrac{\partial \vartheta_1}{\partial y_1} = 0 \end{cases} \quad (2)$$

Here $\mathrm{Re} = \dfrac{Uh}{\nu}$ – Reynolds number; $EL = \dfrac{\nu\lambda}{h^2}$ –elasticity coefficient; $\mathrm{Re}\,\delta$, $\delta = \dfrac{h}{L} = \dfrac{V}{U}$ – quantities are infinitely small quantities; But EL – is an ordinal number of elasticity coefficient.

In this case, the product of velocities over time is preserved. The given equations (2) are written in a systematic "stresses", so in solving specific problems it is necessary to include additional connections between stresses and deformations and their time products tensor components. In general, the system of equations above (2) has the properties of viscosity and elasticity, which are determined by the system of equations (1). These connections (1) are reduced to a dimensional view using substitutions [14]:

$$\begin{cases} \tau_{1,ij} = \displaystyle\sum_{k=1}^{\infty}\tau_{1,k,ij}, \quad \dfrac{\partial \tau_{1,k,ij}}{\partial t_1} + \dfrac{g_k}{\lambda_{1,k}}\tau_{1,k,ij} = 2p_{1,k}B_{i,j}, \\ \dfrac{\partial P_{1,k}}{\partial t_1} + \dfrac{g_k}{\lambda_{1,k}}p_{1,k} = \dfrac{\eta_{1,k}}{\lambda^2_{1,k}}f_k, \\ B_{11} = \dfrac{\partial u_1}{\partial x_1}\dfrac{h}{L}, \quad B_{22} = \dfrac{\partial \vartheta_1}{\partial y_1}\dfrac{h}{L}, \\ B_{21} = B_{12} = \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial y_1} + \dfrac{\partial \vartheta_1}{\partial x_1}\dfrac{h^2}{L^2}\right). \end{cases} \quad (3)$$

By subtracting the terms infinitely small quantities from the system of equations (2) and (3) we obtain a system of simplified equations.

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = -\dfrac{\partial p_1}{\partial x_1} + \dfrac{\partial \tau_{1,21}}{\partial y_1}, \quad \dfrac{\partial p_1}{\partial y_1} = 0, \\ \dfrac{\partial u_1}{\partial x_1} + \dfrac{\partial \vartheta_1}{\partial y_1} = 0, \\ \tau_{1,21} = \sum\limits_{k=1}^{\infty} \tau_{1,k,21}, \quad \dfrac{\partial \tau_{1,k,21}}{\partial t_1} + \dfrac{g_k}{\lambda_{1,k}} \tau_{1,k,21} = 2 p_{1,k} B_{21}, \\ \dfrac{\partial p_{1,k}}{\partial t_1} + \dfrac{g_k}{\lambda_{1,k}} p_{1,k} = \dfrac{\eta_{1,k}}{\lambda_{1,k}^2} f_k, \quad B_{21} = \dfrac{1}{2} \dfrac{\partial u_1}{\partial y_1}. \end{cases} \tag{4}$$

From this we move on to dimensional variables. In this case, a system of equations is formed from equations (4):

$$\begin{cases} \dfrac{\partial u}{\partial t} = -\dfrac{1}{\rho} \dfrac{\partial p}{\partial x} + \dfrac{1}{\rho} \dfrac{\partial \tau_{21}}{\partial y_1}, \quad \dfrac{\partial p}{\partial y} = 0, \\ \dfrac{\partial u}{\partial x} + \dfrac{\partial \vartheta}{\partial y} = 0, \\ \tau_{21} = \sum\limits_{k=1}^{\infty} \tau_{k,21}, \quad \dfrac{\partial \tau_{k,21}}{\partial t} + \dfrac{g_k}{\lambda_k} \tau_{k,21} = p_k \dfrac{\partial u}{\partial y}, \\ \dfrac{\partial p_k}{\partial t} + \dfrac{g_k}{\lambda_k} p_k = \dfrac{\eta_k}{\lambda^2{}_k} f_k. \end{cases} \tag{5}$$

By performing similar substitutions in the cylindrical coordinate system, we can give a system of equations representing the flow of non-Newtonian fluid in long pipes with a circular cross section.

$$\begin{cases} \rho \dfrac{\partial \vartheta_x}{\partial t} = -\dfrac{\partial p}{\partial x} + \dfrac{1}{r} \dfrac{\partial}{\partial r}\left(r \tau_{rx}\right), \quad \dfrac{\partial p}{\partial r} = 0, \\ \dfrac{\partial \vartheta_x}{\partial x} + \dfrac{1}{r} \dfrac{\partial}{\partial r}\left(r \vartheta_r\right) = 0, \\ \tau_{rx} = \sum\limits_{k=1}^{\infty} \tau_{k,rx}, \quad \dfrac{\partial \tau_{k,rx}}{\partial t} + \dfrac{g_k}{\lambda_k} \tau_{k,rx} = p_k \dfrac{\partial \vartheta_x}{\partial r}, \\ \dfrac{\partial p_k}{\partial t} + \dfrac{g_k}{\lambda_k} p_k = \dfrac{\eta_k}{\lambda^2{}_k} f_k. \end{cases} \tag{6}$$

In order to complete the mathematical formulation of the problem of non-stationary and pulsating flows of elastic viscous incompressible fluid in pipes and channels, it is necessary to determine the initial and boundary conditions. The fluid does not move in the initial state, i.e. it maintains a calm state

$$t = 0 \ \ at \quad \vartheta_x = 0, \ \vartheta_r = 0, \quad \frac{\partial p}{\partial x} = 0\,, \tag{7}$$

At non-zero values of time, the current moves under the influence of a constant pressure gradient.

$$t > 0 \;\; at \;\;\; \vartheta_x \neq 0, \;\; \vartheta_r \neq 0, \qquad \frac{\partial p}{\partial x} \neq 0, \tag{8}$$

Given that the flow in question takes place in long channels, then the ratio $\frac{h}{L} << 1$ is satisfied. In addition, using the possibility of expressing in the form of complex functions the oscillating pressure changes in the initial and lower sections of the pipe and channel in the same form, we present the boundary conditions in the following form:

$$\begin{aligned} &x = 0,\; t > 0 \;\; at \;\; p(x,r,t) = p_0^0 + \textstyle\sum_{k=1}^{N} A_k \exp(i\sigma_k t) \;\; , \\ &x = L,\; t > 0 \;\; at \;\; p(x,r,t) = p_L^0 + \textstyle\sum_{k=1}^{N} B_k \exp(i\sigma_k t) \;\; ; \end{aligned} \tag{9}$$

Here σ_k – oscillation frequency in pulsating current; A_k, B_k – constant complex Fure coefficients (they determine the amplitude of the pressure oscillation); p_0^0, p_L^0 these are average pressures in the initial and lower sections at $t = 0$.

Thus, the flow of non-Newtonian fluids with relaxation properties in pipes and channels was mathematically formed. These issues generalize the research work of Womersley [12-15], Pedli [7], Navruzov [9-11] and others, previously known for a one-dimensional case in a certain sense. The system of equations (5) and (6) shows a system of nonlinear equations with respect to the rheology of fluids, as f_k and g_k are determined using the models Meister (M), Berda-Carro (BC), McDonald-Berda-Carro (MBK), and is a function of the shear deformation rate. Therefore, the system of equations (5), (6) has no analytical solutions, so it can be limited to a linear approximation, assuming $f_k = 1,\; g_k \approx 1$ in simplified concrete problems. In this case, the quantities 1 are defined as follows:

$$\eta_k = \frac{\eta}{\xi(\alpha) k^{\alpha}}, \;\; \lambda_k = \frac{\lambda}{k^{\alpha}}.$$

Here η_k *- coefficient of fluid viscosity in stationary flow;* λ_k *- relaxation time;* $\xi(\alpha)$ *- Riman zeta function.*

Thus, a fully linearized form of the system of equations representing the flow of elastic viscous fluid in flat channels and circular cylindrical tubes was obtained, and it was possible to obtain an analytical solution for small amplitude oscillations.

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