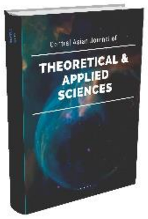

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Fur'e Method for Solving Boundary Value Problems Placed in Parabolic Type Equations

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Annotation: *In this article, using the Maple mathematical package, the solution of the equation of heat conduction in a rod and the equations of heat conduction in a semi-straight line is presented using the Fourier method, that is, the method of separation of variables.*

Keywords: *maple, equation, parabola, Fourier, method, homogeneous, diffusion, example, problem.*

INTRODUCTION.

The process of spreading heat in a straight line, plane and space, as well as the phenomenon of diffusion, which is important in our daily life, is studied through parabolic equations. For these equations, the boundary and Cauchy problems, like the wave equation, allow one-valued separation of the solution of the equation, and they are selected based on the specified mode.

In this article, the solution of the Cauchy problem, defined as the limiting case of mixed problems posed on a finite-length stem, to the equation of heat diffusion in an unbounded-length stem, is found using the same change of variables used in solving boundary value problems for hyperbolic equations, or the Fourier method, and the solution is Poisson we learned that it can be described in an integral form called an integral [1-5].

In this article, using the Maple mathematical package, it is presented to solve the equation of heat conduction in a rod and the equations of heat conduction in a semi-straight line using the Fourier method, that is, the method of separation of variables. The process of solving parabolic equations using the Maple package is described using interesting examples to teach the rules. The application of the Maple package to solving each type of problem is presented sequentially, that is, examples of solving parabolic type equations are described as follows: the calculation formula, analytical and numerical solution, as well as a two-dimensional animated graph of the solution are described, in addition to A two-dimensional graph of several moments of time is depicted for these examples.

RESULT AND DISCUSSION.

The main essence of this subject is the application of Fourier's method to the equation of heat propagation, which is used in solving the boundary or mixed problem that we put in the wave equation. In the previous topics, we got acquainted with the introduction of three types of boundary problems to the heat diffusion equation and the solution of the problem of the uniqueness of their solution [6-12].

We will first familiarize ourselves with the mentioned method on the example of a homogeneous type 1 boundary value problem placed on the homogeneous equation of heat, that is, we will solve the following problem:

$$u_t = a^2 u_{xx},\ 0 < x < \ell, t > 0 \tag{1}$$

of the heat transfer equation

initial and

$$u(x,0) = \varphi(x),\ 0 \le x \le \ell \tag{2}$$

$$u(0,t) = 0,\ \ u(\ell,t) = 0,\ \ t \ge 0 \tag{4}$$

satisfying the boundary condition and $0 \le x \le l$, $t \ge 0$ it is required to find a continuous solution to the second order defined in the field. In this φ (x) is a given continuously differentiable function, φ (0) = 0 satisfies the condition [13-20].

Assuming that there is an infinite solution of the sought form of this problem

$$u(x,t) = X(x)T(t) \ne 0 \tag{5}$$

we look for it in appearance. If we find the necessary particular derivatives of the solution and put them in equation (1), it will be equal to the following equation:

$$X(x)T'(t) = a^2 X''(x)T(t),\ 0 < x < \ell, t > 0.$$

Just like in the wave equation, it doesn't take both sides of the equation a^2 $X(x)$ $T(t)$ we arrive at the following equations:

$$\frac{X''(x)}{X(x)} = \frac{T'(t)}{a^2 T(t)} = -\lambda\ .$$

This equation breaks down into two ordinary differential equations, as we have seen before:

$$\left.\begin{array}{l} X''(x) + \lambda X(x) = 0 \\ T'(t) + \lambda a^2 T(t) = 0 \end{array}\right\} \qquad (5)$$

(1.3.3) boundary conditions take the following form:

$$\left.\begin{array}{l} u(x,0) = 0 \\ u(x,\ell) = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} X(0)T(t) = 0 \\ X(\ell)T(t) = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} X(0) = 0 \\ X(\ell) = 0 \end{array}\right\},$$

because if $T(t) = 0$, based on (4), we arrive at the solution $u(x,t)$ equal to exactly zero. This is not possible according to the conditions [21-26].

So we $X(x)$, $0 \le x \le l$ we come to the following additional problem for the function:

$$X''(x) + \lambda X(x) = 0 \qquad (6)$$

The equation

$$X(0) = 0,\ X(l) = 0 \qquad (7)$$

it is necessary to find a solution that satisfies the conditions. Usually, this problem is called the Sturm-Liouville problem, which corresponds to the homogeneous type 1 boundary value problem imposed on the heat diffusion equation. Equation (6) will have an infinite solution λ the value of Sturm-Liouville problem has a characteristic value and does not correspond to it $X(x)$ and the solution is called the characteristic function corresponding to it [27-32].

Sturm-Liouville problem solution $X(x) = Ce^{kx}$ we look for it in appearance. Then it is called the characteristic equation of the second-order simple linear differential equation (6)

$$k^2 + \lambda = 0 \qquad (8)$$

we arrive at Eq. This is the solution of the quadratic equation λ depends on the sign of its value (negative, zero or positive). That is why we will consider these three cases separately.

1- case. $\lambda<0$ let it be negative. In this case, equation (8) has two distinct reals $k_{1,2}=\pm\sqrt{-\lambda}$ has roots and is the general solution of equation (6).

$$X(x) = Ae^{\sqrt{-\lambda}\,x} + Be^{-\sqrt{-\lambda}}$$

it will be visible. Here, we choose the coefficients A and B in such a way that the boundary conditions (7) are fulfilled. Based on these conditions, we arrive at the following system of linear equations: [1-2]

$$\left.\begin{array}{l} X(0) = 0 \\ X(\ell) = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} A + B = 0 \\ Ae^{\sqrt{-\lambda}\ell} + Be^{-\sqrt{-\lambda}\ell} = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} A = -B \\ B(e^{\sqrt{-\lambda}\ell} - e^{-\sqrt{-\lambda}\ell}) = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} A = 0 \\ B = 0 \end{array}\right\}.$$

The last equality λ = 0 has been (λ ≠ 0 enough to be) $e^{\sqrt{-\lambda l}} \neq e^{-\sqrt{-\lambda l}}$ written based on. So λ < 0 however, the Sturm-Liouville problem has only zero solutions and does not have eigenvalues and eigenfunctions. Now let's look at the second case.

2- case. λ = 0 let it be In this case, equation (6) $X''(x) = 0$ takes the form. Its general solution $X(x) = Ax + B$ it will be visible. Here too, we choose the coefficients A and B in such a way that the boundary conditions (7) are fulfilled: [3-5]

$$\left.\begin{matrix} X(0)=0 \\ X(\ell)=0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} B=0 \\ A\ell=0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} A=0 \\ B=0 \end{matrix}\right\}.$$

So λ = 0 even so, the Sturm-Liouville problem has only a zero solution and does not have an eigenvalue and an eigenfunction. Now let's look at the last case.

3- case. λ > 0 let it be positive. In this case, equation (8) is two joint complexes $k_{1,2} = \pm\sqrt{-\lambda}$ has roots and is the general solution of equation (6)

$X(x) = A\cos\sqrt{\lambda}\,x + B\sin\sqrt{\lambda}\,x$

it will be visible. In this case, we choose arbitrary coefficients A and B in the general solution in such a way that the boundary conditions (7) are fulfilled. Based on these conditions, we arrive at the following system of linear equations:

$$\left.\begin{matrix} X(0)=0 \\ X(\ell)=0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} A=0 \\ B\sin\sqrt{\lambda}\ell=0 \end{matrix}\right\}.$$

From the last system $X(x) \neq 0$ from being (*B=0* if *X(x)=0* would be)

$$\lambda = \lambda_n = \left(\frac{n\pi}{\ell}\right)^2, n = 1,2,3,\cdots$$

and that the most appropriate solution is the constant multiplicative accuracy

$$X_n(x) = sin\frac{n\pi}{l}x$$

So $\lambda = 0$ the Sturm-Liouville problem is positive $\lambda_n = \left(\frac{n\pi}{l}\right)^2, n = 1,2,3\ldots$ xos values and corresponding to them $X_n(x) = sin\frac{n\pi}{l}x$ has eigenfunctions, and eigenfunctions are for scalar multiplication

$$(X_n, X_m) = \int_0^\ell \sin\frac{n\pi}{\ell}x\sin\frac{m\pi}{\ell}xdx = \frac{1}{2}\left(\frac{1}{n-m}\sin(n-m)\pi - \frac{1}{n+m}\sin(n+m)\pi\right) = 0$$

satisfies the condition of equality, i.e. orthogonality.

So (1.3.1) equation only $\lambda_n = \left(\frac{n\pi}{l}\right)^2, n = 1,2,3 \ldots$ only when there is a solution.

$\lambda = \lambda_n = \left(\frac{n\pi}{l}\right)^2, n = 1,2,3 \ldots$ the solution of the second equation of the system is described as follows:

$$T_n(t) = C_n e^{-a^2 \lambda_n t}. \qquad (9)$$

Then according to (4)

$$u_n(x,t) = T_n(t) X_n(x) = C_n e^{-\left(\frac{n\pi}{\ell} a\right)^2 t} \sin\frac{n\pi}{\ell} x.$$

(1) is a linear differential equation and its convergence is currently unknown

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty} C_n e^{-\left(\frac{n\pi}{\ell} a\right)^2 t} \sin\frac{n\pi}{\ell} x$$

Since the series consists of solutions from each of them, the sum is also a solution of equation (1) satisfying boundary conditions (3). [1-3]

CONCLUSION.

In conclusion, a new class of the problem of integral geometry in the lane (corridor) based on parabolas has been studied.

The main results are:

- the analytical view of the solution is obtained;
- the existence theorem of the solution is proved;

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